Drift Limitations in a Shear Wall Considering a Cracked Section

Michel Farid Khouri
Department of Civil Engineering, Branch II, Lebanese University, Roumieh, Lebanon
(email: mkhuri@ul.edu.lb)

Abstract
Literature shows a considerable uncertainty and large differences between seismic codes in evaluating the maximum allowable drift in shear walls. A review of the maximum allowable drift presented in some selective seismic codes show that values range from $h/50$ to $h/2000$, where $h$ is the story height. This difference has created a question among engineers as to which value is correct and what value should be used when evaluating the allowable drift. The purpose of this article is to use structural dynamics and finite element method along with reinforced concrete design to relate the maximum drift at the top to the maximum allowable strain at the bottom of a shear wall and consequently obtain a formula that can be used to determine the maximum allowable drift in a shear wall taking into consideration the effect of a cracked reinforced concrete section. Evaluation of the results show that the formula suggested in this article provides average values between the conservative results of the French code PS92 and the high drift results of UBC and IBC.

1 Introduction
Shear walls are mainly constructed to support lateral forces due to wind or earthquakes. Under these loads, the structure will have a lateral displacement (known as Drift) the magnitude of which is defined by the movement of the lateral load resisting elements (shear walls). The question remains: what is
the allowable drift of a given shear wall? And consequently what is the maximum allowable drift in a building? Many investigators have suggested values for maximum displacement at the top of buildings, and maximum values for inter-story drift. These values range from taking \( h/50 \) where \( h \) is the height of the building as suggested by The Uniform Building Code [1] and International Building Code [2] to choosing about \( h/2000 \) as in The Lebanese Code [3].

This study makes use of the finite element method and structural dynamics presented in Bathe [4] and Craig [5] to evaluate the displacement at the top of a shear wall and suggests an equation that can be followed in determining the maximum allowable drift.

In a previous similar study done by the author on the evaluation of the maximum allowable drift, results have shown that drift values are relatively low if the shear wall is considered to be a full un-cracked section, Khouri [6].

In this study a cracked section is considered as per UBC and ACI recommendations and the obtained results are comparative to the seismic codes used in earthquake design. The shear wall is assumed to be a vertical beam in flexure with constant stiffness throughout its height. The maximum strain at the \( i^{th} \) story along the shear wall is then computed using simplified hand calculated finite element method, and the results are combined to obtain a relation between the maximum displacement in a shear wall and the maximum strain at the bottom of the shear wall after considering a cracked section.

Consequently, an expression for the maximum allowable drift is obtained and comparison between the suggested formula and various seismic codes and investigators is then done.

### 2 Background

Many investigators and seismic codes suggest values for maximum allowable story drift or maximum allowable displacement at the top of a building but these values differ significantly. A comparison of some selective values is presented in Table 1 of Reference [6]; also the reader is referred to References [1-3] and [7-11].

In observing the drift values, it is clear that the range vary between \( h/50 \) and \( h/2000 \). The questions that can be asked at this point are: why are these differences in the estimation of the lateral drift? On what basis these estimations are made? How can we use actual structural behavior to determine maximum allowable drift? The first attempt to quantify the drift based on real structural behavior was done by the author [6]. The following article similarly explains a procedure to estimate the maximum allowable drift between stories and consequently determines the same at the top of a building or a shear wall.

### 3 Matrices Assembly (Shear Building)

The objective is to generate the stiffness matrix for a shear building. To represent a shear wall, a vertical beam in flexure is assumed which means that only the flexural effects of the element are considered and the horizontal displacements are only due to flexure, where the rotation of the nodes are taken equal to zero. Using boundary conditions \( q_3=q_6=0 \) to assume no rotation, and \( F_2=F_5=0 \) to assume no axial force, and replacing in:

\[
\{F\} = [K]\{q\} \Rightarrow
\]

\[
\begin{bmatrix}
F_1 \\
F_4
\end{bmatrix} =
\begin{bmatrix}
EI & -12 & -12 \\
-12 & 12 & 12
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_4
\end{bmatrix}
\]

From Figure 2, eqn (3) can be assembled relative to degrees of freedom 4, 7, …, \( 3j+1 \)…
Also constant stiffness is assumed in all stories where \( k^1 = k^2 = \ldots = k^{(N)} = k \).

**Figure 1: Representation of an N-story building.**

The system of \( N \) degrees of freedom will be of the following form:

\[
\begin{bmatrix}
F_4 \\
F_7 \\
\vdots \\
F_{3j+1} \\
F_{3(N+1)+1} \\
F_{3N+1}
\end{bmatrix} \begin{bmatrix}
2 & -1 & 0 & \ldots & \ldots & 0 & 0 \\
1 & 2 & -1 & 0 & \ldots & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
1 & 0 & 0 & \ldots & 0 & -1 & 2 \\
0 & \ldots & 0 & \ldots & \ldots & 0 & -1 \\
0 & \ldots & 0 & \ldots & \ldots & 0 & \ldots
\end{bmatrix} \begin{bmatrix}
q_4 \\
q_7 \\
\vdots \\
q_{3j+1} \\
q_{3(N+1)+1} \\
q_{3N+1}
\end{bmatrix} = \begin{bmatrix}
2 -1 0 \ldots \ldots 0 0 \\
1 2 -1 0 \ldots \ldots 0 \\
\vdots \\
1 \ldots 0 \ldots 0 -1 2 -1 \\
0 \ldots 0 \ldots \ldots 0 -1 2 -1 \\
0 \ldots 0 \ldots \ldots 0 -1 2 -1
\end{bmatrix} \begin{bmatrix}
F_4 \\
F_7 \\
\vdots \\
F_{3j+1} \\
F_{3(N+1)+1} \\
F_{3N+1}
\end{bmatrix}
\]

Where, \( k \) is the stiffness of one story

\[
F_i = \frac{2iV}{N(N+1)}, \text{and } F = k.q \Rightarrow q = k^{-1} F,
\]

where,

\[
q_{3N+1} = \Delta = \frac{2V}{M(N+1)} \frac{1}{k} \left( \frac{i}{1+2+3+\ldots+i+N} \right) \]

\[
\Delta = \frac{2V}{N(N+1)} \frac{1}{k} \sum_{i=1}^{N} i^2, \text{but}
\]

\[
\sum_{i=1}^{N} i^2 = N(N+1)(2N+1) \frac{6}{3}
\]

Therefore, \( \Delta = \frac{(2N+1)V}{3} \frac{1}{k} \).

The formula for the base shear becomes

\[
V = \frac{3}{2N+1} k \Delta
\]

**5 Displacement / Relative Displacement**

The displacement \( q_i = (V/k) X_i \), and let:

\[
X = \sum_{i=1}^{N} i = \frac{N(N+1)}{2}, \Rightarrow
\]

\[
q = k^{-1} F
\]

where \( X_i \) is computed as follows:

\[
X_j = \frac{i-1}{X} \sum_{j=1}^{i-1} j^2 - \frac{i-1}{X} \sum_{j=1}^{i-1} j
\]

\[
\Rightarrow X_j = \frac{i-1}{X} \sum_{j=1}^{i-1} (j^2 - j), \text{where},
\]

\[
\sum_{j=1}^{i-1} j = \frac{i}{2} (i-1)(i), \text{and},
\]

\[
\sum_{j=1}^{i-1} j^2 = \frac{i}{6} (i-1)(2i-1)
\]

\[
X_j = \frac{1}{X} \left[ 1 - \frac{(i^2 - i)}{6} \right]
\]

Therefore, the formula that gives the
displacement \( q_i \) at the \( i^{th} \) story is,

\[
q_i = i \left[ 1 - \left( \frac{2}{6X} \right)^2 \right] \times \frac{V}{k} \quad \tag{7}
\]

\[
q_i - q_{i-1} = \left( \frac{2X - i^2 + i}{2X} \right) \left( \frac{V}{k} \right) \quad \tag{8}
\]

6 Strain Determination Using FEM

\( U(x) \) represents the deflection at \( x \). At \( x=0, U=q_1 \) and \( U'=q_2 \) and at \( x=L, U=q_3 \) and \( U'=q_4 \).

If \( f \) represents the shape function, the deflection function is, \( U = f_1q_1 + f_2q_2 + f_3q_3 + f_4q_4 \), where,

\[
f_1 = 1 - 3\phi^2 + 2\phi^3, \quad f_2 = L \phi (1-\phi)^2, \quad f_3 = \phi^2 (3-2\phi), \quad \text{and} \quad f_4 = L \phi^2 (\phi-1).
\]

Now, \( \phi = x/L \) (\( L \) is the length of the element, height of one story on the shear wall.) If we have a shear building \( \Rightarrow q_2 = q_4 = 0 \) \( \Rightarrow U = (1 - 3\phi^2 + 2\phi^3) q_1 + \phi^2 (3-2\phi) q_3 \),

\[
\varepsilon_x = \frac{\partial u}{\partial x}, \quad \text{and} \quad \frac{dU}{dx} = 0 \quad \text{Where:}
\]
- \( u \) is the displacement in the \( x \) direction, \( u = -y \frac{dU}{dx} \).
- \( y \) is the algebraic distance measured from the neutral axis to the extreme fiber of the shear wall. (N.B. maximum \( \varepsilon_i \) is at maximum \( y \))
- \( x \) is the abscissa along the shear wall between the levels (i-1) and \( i \).
- \( \varepsilon_x \) is the strain in \( x \) direction.
- \( \varepsilon_x \) is the angle of rotation of the section of the shear wall at level \( x \).

The strain \( \varepsilon \) can be computed as,

\[
\varepsilon_x = \frac{\partial u}{\partial x} = -y \frac{\partial^2 U}{\partial x^2}
\]

If \( y \) is considered to be positive in the opposite direction of \( U \), \( \Rightarrow \)

\[
\varepsilon_x = y \frac{\partial^2 U}{\partial x^2} = B q_i f_i f_2 f_3 f_4, \quad \text{and} \quad \varepsilon = B q_i
\]

7 Maximum Drift in a Shear Wall

The maximum displacement at the top of the shear wall is reached when the reinforcement strain in the tension zone at the lowest section of the shear wall is equal to \( \varepsilon_{st} \) (maximum allowable strain in steel), and the strain in the extreme fiber of the compression zone in the same section is equal to \( \varepsilon_c = \) maximum strain limit of concrete in compression = 0.003.

So the lowest section in the shear wall, which is the most critical section, is considered to have a triangular distribution.
From eqn(11), $\varepsilon_{\text{max}} = \frac{6}{k} \frac{V}{L^2}$, and from eqn(5), $V = \frac{3}{2N + 1} k \Delta \Rightarrow \frac{V}{k} = \frac{3}{2N + 1} \Delta$.

$\Rightarrow \varepsilon_{\text{max}} = \frac{6}{L^2} \frac{k}{2N + 1} \Delta = \frac{18y}{L^2(2N + 1)}$ \hspace{1cm} (12)

where D is the maximum displacement at the top of the shear wall. From similar triangles of the section:

$$\frac{y}{\varepsilon_{\text{st}}} = \frac{x}{\varepsilon_{\text{c}}} \Rightarrow y = \frac{\varepsilon_{\text{st}}}{\varepsilon_{\text{c}}} d.$$ \hspace{1cm} (13)

The maximum strain in the shear wall presented in eqn (12) at the level of steel should be smaller than $\varepsilon_{\text{st}}$:

$$\varepsilon_{\text{max}} = \frac{18y}{L^2(2N + 1)} \Delta \leq \varepsilon_{\text{st}}$$

replace y from eqn(13) \hspace{1cm} \Rightarrow \hspace{1cm} \Delta \leq \frac{L^2(2N + 1)(\varepsilon_{\text{c}} + \varepsilon_{\text{st}})}{18(d)} \hspace{1cm} (14)

If the strain in the steel is to stay below $\varepsilon_y$ (yielding strain), the maximum allowable drift at the top of the shear wall is,

$$\Delta \leq \frac{L^2(2N + 1)(\varepsilon_{\text{c}} + \varepsilon_y)}{18(d)} \hspace{1cm} (15)$$

In this case, the lowest section of the shear wall behaves as a balanced section; the limits are reached in the reinforcement in tension and in the concrete in compression at the same time, and at that point, the maximum allowable drift at the top of the shear wall is reached.

### 7.1 Cracked section consideration

According to UBC97, Modeling Requirement section 1630.1.2 “Stiffness Properties of Reinforced Concrete and masonry elements shall consider the effects of cracked sections”. This means that when a designer uses cracked section analysis then according to ACI 318-08 [13], section 10.10.4.1, the shear wall moment of inertia would have to be reduced by a factor 0.35 such that the inertia would become 0.35 I. This will consequently reduce the stiffness of the lateral load resisting elements and will equally increase the drift by the same factor, which means that the drift should be multiplied by a factor of $(1/0.35)$ to get a value taking into consideration the effects of a cracked section suggested by UBC-97 and ACI-318.

$$\Delta \leq \left(\frac{1}{0.35}\right) \frac{L^2(2N + 1)(\varepsilon_{\text{c}} + \varepsilon_y)}{18(d)}$$ \hspace{1cm} (16)

Notice that, in this formula, as d increases the maximum allowable displacement decreases since any small movement tends to cause larger strain at the bottom section of the shear wall. On the other hand, and as far as maximum displacement is concerned and disregarding economical and architectural issues, it is better to use more number of shear walls with small d than to use fewer shear walls with large d; keeping in mind that the inertia of a shear wall is increased cubically as a function of d, and a bigger d will increase the stiffness significantly in a certain direction.

### 8 Example and Comparison

In comparing formula (16) suggested by the author with other drift values, take the effective depth of the shear wall $d = 2m$, the inter-story length $L = 3m$; also from Figure 2, if $\varepsilon_{\text{st}} = \varepsilon_y = 0.00207$ and $\varepsilon_{\text{c}} = 0.003$, the formula becomes $\Delta \leq \frac{(H_t + 1.5)}{414.2}$, where $H_t$ is the total height of the building.

In comparing the formula suggested by the author with the maximum allowable drift suggested by other investigators as shown in Figure 3, it can be observed that the allowable displacement (drift) suggested by IBC and...
UBC are much greater than the others. Also Fintel [14] suggested \((h/500)\), while Searer et. al. [15,16] suggested \((h/100)\). These values are much higher than what is suggested by PS92, and the Lebanese Code.

![Image](image1.png)

*Figure 2: Typical Stress-Strain Curves for Reinforced Bars 9 [17].*

In addition, if the allowable strain in steel \(\varepsilon_{st}\) is considered to be larger than \(\varepsilon_y\) (which means that yielding of steel is permitted, or in other words the strain is beyond yield on the yield plateau as in Figure 3 (b)), a larger allowable inter story and overall drift will be permitted. This may be the reason why some codes suggest a larger allowable drift than eqn (16); the structure is allowed to pass the elastic limits and displace within the plastic region taking into consideration dynamic reversals under wind or earthquake loading.

It is also important to note that the formula suggested by the author also provides a good approximation of the drift values for strains that go beyond the elastic limits in the reinforcing steel.

9 Conclusion

In this study, shear building was analyzed using the finite element method. The shear was obtained as a function of the displacement. A value for the displacement at any story was obtained, and from which a function for the relative displacement between two stories was then determined. Using the above, an equation for the maximum strain was resolved. A limiting value for the maximum drift was obtained as a function of the height of a story, number of stories, depth of tension steel \(d\) in a shear wall, the strain of steel \(\varepsilon_{st}\) and maximum allowable concrete strain. The shear building was analyzed like a beam ignoring vertical loads and assuming constant lateral stiffness in all stories. In comparing the results obtained in this work and the results obtained by the previous work done by the author [6], it is clear that shear wall representation is more appropriate when cracked section is considered.
as in this article. Also, comparison between the results of the formula suggested by the author in this work and the maximum allowable drift values suggested by others show that UBC, IBC, the Japanese code and others tend to suggest a higher maximum allowable drift or inter-story drift; on the other hand, the French code (PS92) and the Lebanese code give lower more conservative values than the suggested formula. This means that the formula suggested in this paper gives average values as compared to most codes. The value $h/50$ suggested by UBC97 and IBC 2006 can be considered a high end for a shear building in the sense that it generates large strains at the bottom of a shear wall. Even though high drift values correspond to a flexible structure and lower lateral forces, such large drifts may be dangerous. It is now left for the designing engineer to evaluate his structure and decide/choose a maximum allowable strain limit for concrete and for steel, and determine the corresponding maximum allowable drift values. Finally, the formula suggested in this paper is very useful and gives a very good estimate of the drift. Most of the previous work done on drift by other investigators has given values for the drift based on experience, judgement and comfort level at higher stories without reference to actual structural behaviour. This paper addresses real structural behaviour.

Acknowledgements
The author thanks Optimal Engineering Consulting and Contracting, (OECC) for sponsoring the major part of this work and special thanks to the Lebanese University, Faculty of Engineering, Branch II for sponsoring part of this on-going research. Also, thanks to the engineers Bassam Mazloum, Johnny Jreige and Gilbert Abou Zeid who helped in this paper.

References


